**Study of the reflection and transmission between the three material by 1D-FDTD PML Final Report**

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**摘要**

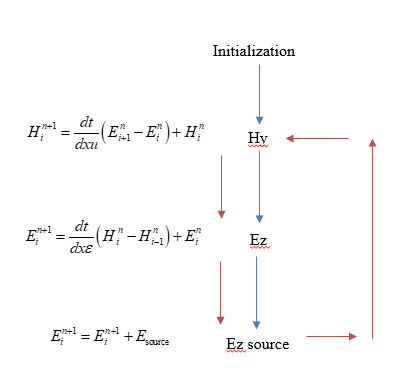
本研究是利用1D-FDTD (Finite-Difference Time-Domain)來模擬並計算三個材料之間的反射係數和穿透係數，邊界的部分則是利用PLM (Perfectly Matched Layer)當作吸收，最後使用Fresnel equation來計算並驗證因為不同的折射係數發生的反射率和穿透率。

**方法**

本程式使用的1D-FDTD是由Maxwell's equations推倒後的電磁場結果



為了模擬同時計算，程式的計算步驟如下:



模擬反射率的計算方法為路徑積分。



為了有更準確的測量，邊界使用PML來吸收，使波可以在兩個材料間傳播而不受到反射波的影響，方程式必須做以下修改:

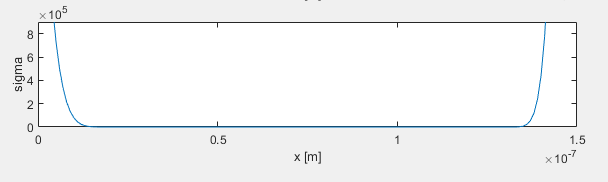


而電導率的設置如下:



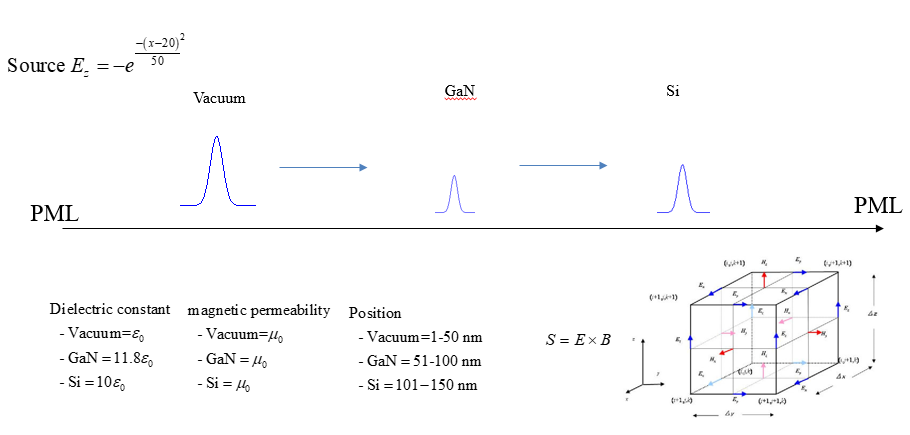
其中為導電率的變化程度，*R*為PML邊界的吸收係數，*w*為PML的寬度，c為光速。

電導率的分布結果圖。



**實驗**

下圖是當模擬高斯波的行徑方向以及材料(Vacuum, GaN以及Si)位置的參數配置。



**驗證**

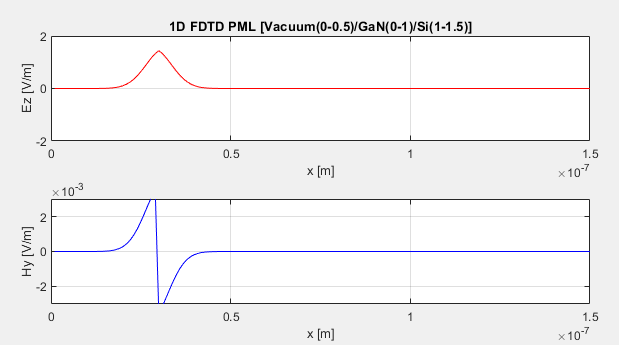
為了驗證模擬值，這裡使用Fresnel equation來驗證模擬。



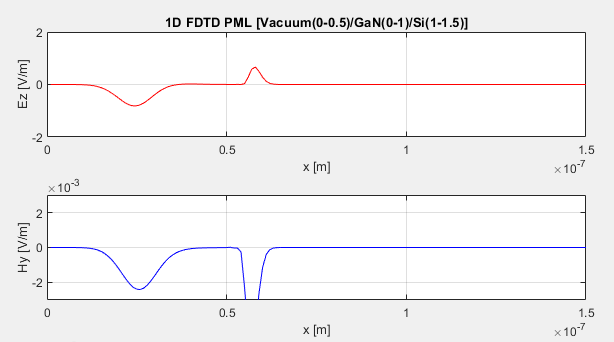
*R*違反射係數，n為折射率，為介電係數，為導磁係數，*T*為穿透率。

**結果**

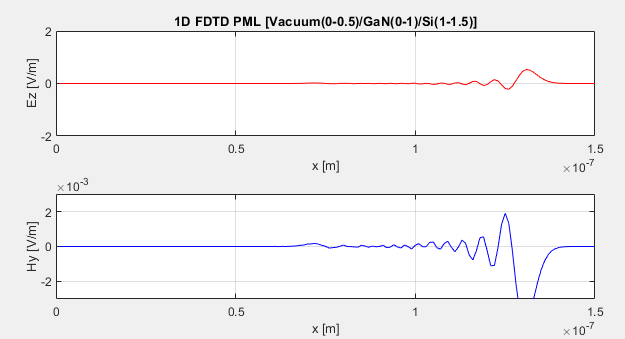
高斯波源位於真空，並開始傳播。



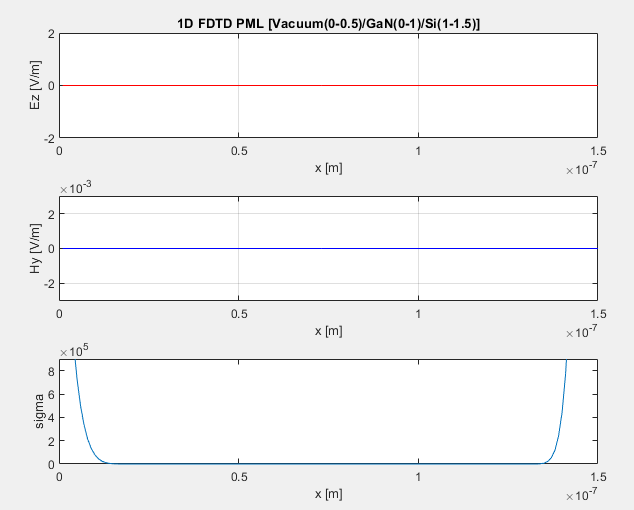
當第一個波從真空傳播到GaN時，因為折射率差異很大，所以有很明顯的反射



而GaN和Si折射率相近，所以反射不明顯。



由於觸碰到PML邊界，所以最後所有的波均被吸收。



利用1D-FDTD的模擬值與Fresnel equation理論值做比較。

|  |  |  |  |
| --- | --- | --- | --- |
| 材料 | *R*模擬值% | *R*理論值% | 誤差% |
| Vacuum - GaN | 80.20% | 29.65% | 170.46% |
| GaN - Si | 10.53% | 0.12% | 8537.8532% |
| n=1 – n=1 | 100% | 100% | 0% |
| n=1 – n=999 | 100% | 88% | 13.6% |

**結論**

由於在非常極端的狀況下(n=1 – n=999)就存在著13.6%的誤差，導致很難在一般的情況下有很準確的模擬值，如果使用較高精度的模擬，也許會對結果會有很大的改善。

MATLAB codes:

clear all

clc

%% dimenstion parameter

xdim=150;

dx=1e-9; % [m]

Steps=1000;

PML\_w=20;

PML\_n=6;

PML\_R=1e-6; % reflection coefficient

%% Souce

source=30;

intensity = 3;

wide = 1;

const = 50;

%%

epsilon0=8.85e-12;

u0=1.2566e-6;

c=3e8;

dt=dx/c;

Ez=zeros(1,xdim);

Hy=zeros(1,xdim);

Ez\_g=zeros(1,xdim);

Hy\_g=zeros(1,xdim);

globel\_Hy = zeros(Steps,xdim);

globel\_Ez = zeros(Steps,xdim);

%% three materials

epsilon=epsilon0\*ones(1,xdim);

u=u0\*ones(1,xdim);

a\_meterial = [1,50];

b\_meterial = [51,100];

c\_meterial = [101,150];

epsilon(1,a\_meterial(1):a\_meterial(2))=1\*epsilon0;

epsilon(1,b\_meterial(1):b\_meterial(2))=11.5\*epsilon0;

epsilon(1,c\_meterial(1):c\_meterial(2))=10\*epsilon0;

u(1,a\_meterial(1):a\_meterial(2))=1\*u0;

u(1,b\_meterial(1):b\_meterial(2))=1\*u0;

u(1,c\_meterial(1):c\_meterial(2))=1\*u0;

record = [80,300]; %caculation reflection (simulaiton) [80,330]

%% PML

% Ez conductivity

PML\_maxsigma=(-log10(PML\_R)\*(PML\_n+1)\*epsilon0\*c)/(2\*PML\_w\*dx);

PML\_boundary\_l=((epsilon(1,PML\_w)/epsilon0)\*PML\_maxsigma)/((PML\_w^PML\_n)\*(PML\_n+1));

PML\_boundary\_r=((epsilon(1,xdim-PML\_w)/epsilon0)\*PML\_maxsigma)/((PML\_w^PML\_n)\*(PML\_n+1));

sigma=zeros(1,xdim);

x=0:PML\_w;

for i=1:xdim

sigma(1,PML\_w+1:-1:1)=PML\_boundary\_l\*((x+0.5\*ones(1,PML\_w+1)).^(PML\_n+1)-(x-0.5\*[0 ones(1,PML\_w)]).^(PML\_n+1));

sigma(1,xdim-PML\_w:xdim)=PML\_boundary\_r\*((x+0.5\*ones(1,PML\_w+1)).^(PML\_n+1)-(x-0.5\*[0 ones(1,PML\_w)]).^(PML\_n+1));

end

% Hy conductivity

sigma\_s=(sigma.\*u)./epsilon;

% Hy coefficient

A=((u-0.5\*dt\*sigma\_s)./(u+0.5\*dt\*sigma\_s));

B=(dt/dx)./(u+0.5\*dt\*sigma\_s);

% Ez coefficient

C=((epsilon-0.5\*dt\*sigma)./(epsilon+0.5\*dt\*sigma));

D=(dt/dx)./(epsilon+0.5\*dt\*sigma);

%% 1D-FDTD

for time=1:1:Steps

time;

% time boundary

if time < source-2 % left side

xi=source-time-1;

else

xi=1;

end

if time < xdim-1-source % right side

xf=source+time;

else

xf=xdim-1;

end

% Update Hy from Ez

for i = xi:xf

Hy(i)=A(i).\*Hy(i)+B(i).\*(Ez(i+1)-Ez(i));

end

% Hy source

Hy\_g(time) = intensity\*exp(-((time-source)/wide)^2/const);

% Update Ez from Hy

for i = xi:xf

Ez(i+1)=C(i+1).\*Ez(i+1)+D(i+1).\*(Hy(i+1)-Hy(i));

end

% Ez source

Ez\_g(time) = intensity\*exp(-((time-source)/wide)^2/const);

Ez(source) = Ez(source) + Ez\_g(time);

globel\_Ez(time,:) = Ez(1,:);

globel\_Hy(time,:) = Hy(1,:);

%Reflection

totalE(1,time) = sum(abs(globel\_Ez(time,:)));

totalH(1,time) = sum(abs(globel\_Hy(time,:)));

if time == record(1)

%caculate the simulation value

rE1=sum(abs(globel\_Ez(time,a\_meterial(1):a\_meterial(2))));

RE1 = rE1/totalE(1,time);

tE1=sum(abs(globel\_Ez(time,b\_meterial(1):b\_meterial(2))));

TE1 = tE1/totalE(1,time);

rH1=sum(abs(globel\_Hy(time,a\_meterial(1):a\_meterial(2))));

RH1 = rH1/totalH(1,time);

tH1=sum(abs(globel\_Hy(time,b\_meterial(1):b\_meterial(2))));

TH1 = tH1/totalH(1,time);

%caculation the fresnel value

n1 = sqrt(epsilon(1,a\_meterial(1))\*u(1,a\_meterial(1)));

n2 = sqrt(epsilon(1,b\_meterial(1))\*u(1,b\_meterial(1)));

f\_R1 = ((n1-n2)/(n1+n2))^2;

%error (fresnel-simulation)/fresnel

error\_RE1 = abs(f\_R1-RE1)/f\_R1\*100;

error\_RH1 = abs(f\_R1-RH1)/f\_R1\*100;

string = [' '];

disp(string)

string = ['In first reflection'];

disp(string)

string = ['Ez Simulation value is ', num2str(RE1\*100), '%, fresnel value is ', num2str(f\_R1\*100), '%, error is ', num2str(error\_RE1), '%'];

disp(string)

% string = ['Hy Simulation value is ', num2str(RH1\*100), '%, fresnel value is ', num2str(f\_R1\*100), '%, error is ', num2str(error\_RH1), '%'];

% disp(string)

end

if time == record(2)

%caculate the simulation value

rE2=sum(abs(globel\_Ez(time,b\_meterial(1):b\_meterial(2))));

RE2 = rE2/tE1;

tE2=sum(abs(globel\_Ez(time,c\_meterial(1):c\_meterial(2))));

TE2 = tE2/tE1;

rH2=sum(abs(globel\_Hy(time,b\_meterial(1):b\_meterial(2))));

RH2 = rH2/tH1;

tH2=sum(abs(globel\_Hy(time,c\_meterial(1):c\_meterial(2))));

TH2 = tH2/tH1;

%caculation the fresnel value

n2 = sqrt(epsilon(1,b\_meterial(1))\*u(1,b\_meterial(1)));

n3 = sqrt(epsilon(1,c\_meterial(1))\*u(1,c\_meterial(1)));

f\_R2 = ((n2-n3)/(n2+n3))^2;

%error (fresnel-simulation)/fresnel

error\_RE2 = abs(f\_R2-RE2)/f\_R2\*100;

error\_RH2 = abs(f\_R2-RH2)/f\_R2\*100;

string = [' '];

disp(string)

string = ['In second reflection'];

disp(string)

string = ['Ez Simulation value is ', num2str(RE2\*100), '%, fresnel value is ', num2str(f\_R2\*100), '%, error is ', num2str(error\_RE2), '%'];

disp(string)

% string = ['Hy Simulation value is ', num2str(RH2\*100), '%, fresnel value is ', num2str(f\_R2\*100), '%, error is ', num2str(error\_RH2), '%'];

% disp(string)

end

% plot

subplot(3,1,1)

plot((1:xdim)\*dx,Ez,'color','r');

titlestring=['1D FDTD PML [Vacuum(0-0.5)/GaN(0-1)/Si(1-1.5)]'];

title(titlestring,'color','k');

xlabel('x [m]');

ylabel('Ez [V/m]');

axis([0 xdim\*dx -2 2]);

grid on

subplot(3,1,2)

plot((1:xdim)\*dx,Hy,'color','b');

xlabel('x [m]');

ylabel('Hy [V/m]');

axis([0 xdim\*dx -3e-3 3e-3]);

grid on

subplot(3,1,3)

plot((1:xdim)\*dx,sigma)

xlabel('x [m]');

ylabel('sigma');

axis([0 xdim\*dx 0 900000]);

getframe;

end